Name:____

Water as a Habitat: Episode 5
What's Lurking in the Waters?

A Lionfish Population Investigation

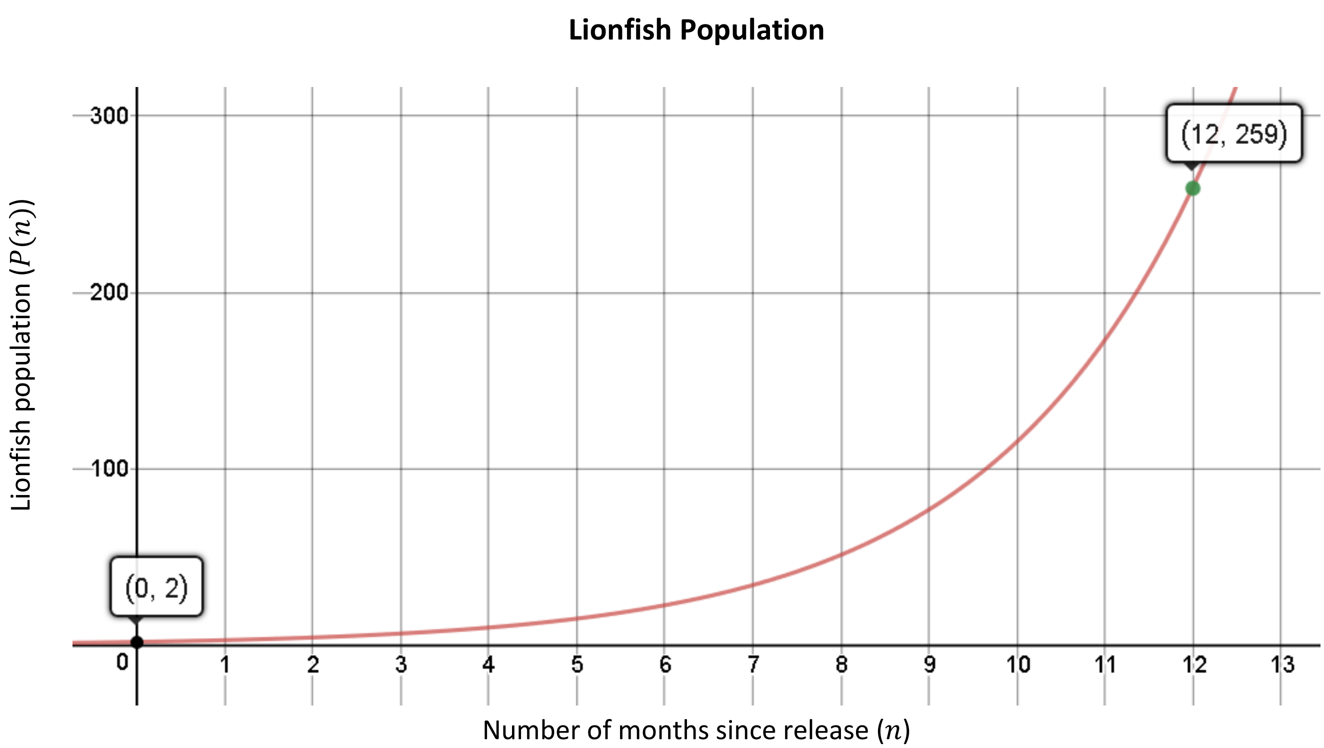
Scenario 1

Suppose the current population of lionfish off of the Treasure Coast was due to the release of 2 overgrown lionfish pets, one male and one female. If the population of lionfish off the coast is growing at a rate of 50% per month, write and graph an exponential function to represent the lionfish population over a one year time period.

Let n be the number of months since the release of the 2 pets
Let $P(n)$ be the total lionfish population.

$P(n) = 2(1.5)^n$

n	0	1	2	...	12
$P(n)$	2	3	5	...	259



A) Is it appropriate to use an exponential model for the population growth of the lionfish? Explain why or why not.
Exponential growth will continue to increase very rapidly. It is not appropriate to model population growth over an unrestricted domain. If the domain is restricted to a specific time period, it can be used. At some point, growth rates would change and the model will become less accurate.

Name:____

B) Write the explicit formula to represent the population growth function of the lionfish as a geometric sequence, where the original number of lionfish is given when $n = 0$, or by the zero term, $P_0 = 2$.
$P(n) = 2(1.5)^n$
Starting the sequence with the zero term allows us to use the same exponential function to model the geometric sequence.

C) How would you find P_{n+1} if you know term P_n?
P_{n+1} would be P_n multiplied by the growth factor of 1.5.

D) Write a recursive formula for P_{n+1} in terms of P_n to represent the population growth function.
$P(n) = (P_{n-1})1.5$

E) Using your explicit formula for the geometric sequence in part B, calculate the total population of the lionfish at the end of the first 2 years.
$P(n) = 2(1.5)^n$
$P(24) = 2(1.5)^{24} = 33{,}668$

F) What is the total change in the number of lionfish over these 2 years?
$\text{Total Change} = 33{,}668 - 2 = 33{,}666$

G) What is the average rate of change of lionfish per month over the 2 years?
$\text{Average Rate of Change} = \frac{33{,}668-2}{24-0} = \frac{33{,}666}{24} = 1{,}402.75 \text{ fish per month}$

H) Use the function to determine the value of n when the population reaches 1 million.

$$1{,}000{,}000 = 2(1.5)^n$$
$$500{,}000 = 1.5^n$$
$$\log_{1.5} 500{,}000 = n$$
$$n = \frac{\log 500{,}000}{\log 1.5} = 32.4 \text{ months}$$

Name:____

<u>**Scenario 2**</u>

A mature female lionfish can release between 10,000 and 30,000 eggs every 4 days. She can do this year round while living in warm, tropical waters. Suppose the number of eggs that are fertilized and survive to become mature lionfish is 8% of the total number of eggs spawned.

A) Determine how many times this lionfish will spawn in one year. Round to the nearest whole number of events.

$n = \frac{365}{4} = 91 \text{ times}$

B) Use the mid-interval value of eggs released to determine how many eggs will develop into lionfish from each spawning event.

$\frac{10{,}000+30{,}000}{2}(0.08) = 20{,}000(0.08) = 1{,}600 \text{ lionfish}$

C) Calculate the number of lionfish that were spawned by a female and living in the surrounding area after 30 days.

$f = 20{,}000(0.0.8)\square\left(\frac{30}{4}\right) = \frac{1{,}600(30)}{4} = 12{,}000$

D) Juvenile lionfish do have a mortality rate, not all of them reach maturity. Suppose for each day after the first 30 days of life, that 15% of these existing juvenile fish are removed from the ecosystem. Some will perish, others will be eaten and others will get trapped and/or be caught by divers.

Let $P(x)$ be the population of lionfish

Let $x = 1$ be the first day the population changes

$P(x) = 12{,}000(0.85)^x$

E) Is this a growth or decay model? Justify your answer.

This is exponential decay because the decay factor, 0.85, is less than one.

F) How many of the juvenile fish remain on the 45th day?

$P(15) = 12{,}000(0.85)^{15} = 1{,}048.25 = 1{,}048 \text{ fish}$